Branching random walks and Gaussian fields

Ofer Zeitouni

Weizmann Institute and University of Minnesota

June 2012

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BRW to GFF

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Lectures I,II - Branching random walks in Z - maximal displacement

Model

- Law of large numbers: comparison with independent walks, first and second moments.
- The Dekking-Host argument and tightness.
- Logarithmic corrections for the mean the lower bound.

Lecture III - Maximal displacement - ct'd

- Logarithmic corrections upper bound.
- Time varying profiles. Phase transitions and non-logarithmic corections.

Lectures IV,V - the 2D Gaussian free field

- Model and Markov property; maxima.
- Basic inequalities: Slepian, Borell-Tsirelson, Sudokov-Fernique.
- LLN for maxima upper bound and approximate tree structure.
- The Dekking-Host argument adapted.
- Expectation of maximum: comparison and modified BRW.
- Analysis of MBRW

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${\mathcal T}$ - tree rooted at o.

|v| - distance of v from root. d_v - degree of v. $D_n := \{v \in V : |v| = n\}$ (*n*th generation). $o \leftrightarrow v$ - vertices/edges on geodesic connecting o and vBRW model $\{X_e\}_{e \in E}$ i.i.d., law μ . $S_v = \sum_{e \in o \leftrightarrow v} X_e$ sum along geodesic (BRW). Maximal displacement:

$$M_n = \max_{v \in D_n} S_v \, .$$

Assumptions (for these lectures):

 μ possesses super-exponential tails:

$$E_{\mu}(e^{\lambda X_e}) =: e^{\Lambda(\lambda)} < \infty, \quad \lambda \in \mathbb{R}.$$

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The tree \mathcal{T} is a *k*-ary tree, with $k \ge 2$: $d_v = k + 1_{v \ne 0}$.

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