Synchronizing Finite Automata

I. History and Motivation

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CSClub, St Petersburg, November 13, 2010

1. Can Modern Math be Understood?

"*Most current mathematical research, since the 60s, is devoted to fancy situations: it brings solutions which nobody understands to questions nobody asked*" (quoted from Bernard Beauzamy, "Real life Mathematic", Irish Math. Soc. Bull. 48 (2002), 43-46).

This provocative claim is certainly exaggerated but it does reflect a really serious problem: a communication barrier between mathematics (and exact science in general) and human common sense.

The barrier results in a paradox: while the achievements of modern mathematics are widely used in many crucial aspects of everyday life, people tend to believe that today mathematicians do "abstract nonsense" of no use at all.

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In many cases there is an inherent difficulty: mathematics is difficult. Normally non-mathematicians do accept the fact that a solution to a mathematical problem may be difficult to digest but the point is that it is usually hard to explain to them why the solution is worth the effort.

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Well, you have proved Fermat's Last Theorem, congratulations!

Will my cow give more milk now?

CSClub, St Petersburg, November 13, 2010

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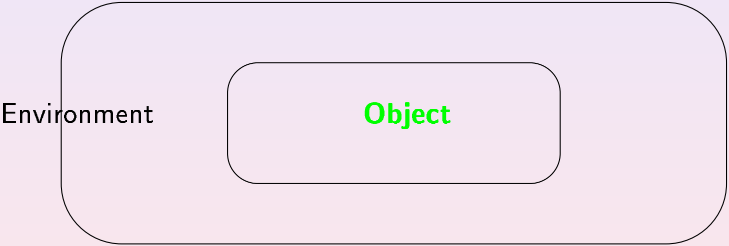

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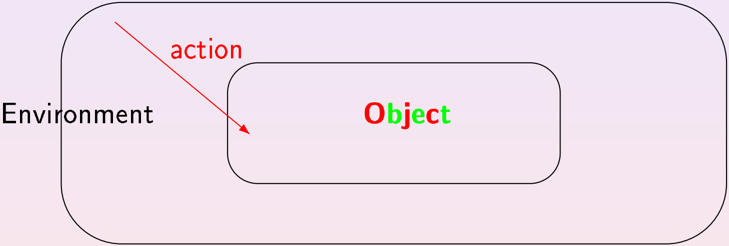

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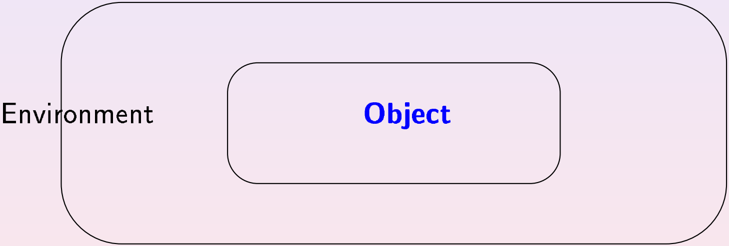

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This notion originates in the seminal work by Alan Turing ("On Computable Numbers, With an Application to the Entscheidungsproblem", Proc. London Math. Soc., Ser. 2, 42 (1936), 230–265).

"The behavior of the computer at any moment is determined by the symbols which he is observing, and his state of mind at that moment".

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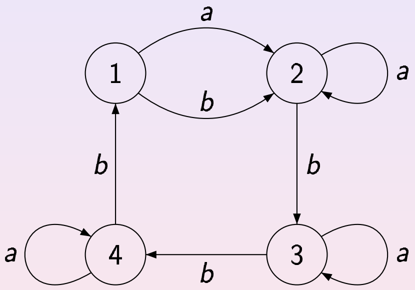

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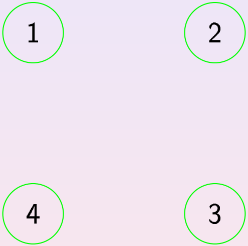

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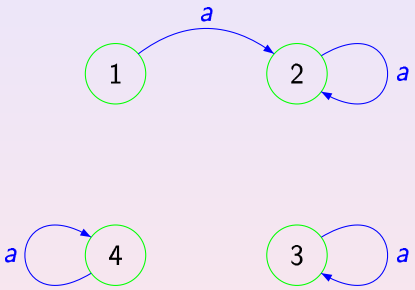

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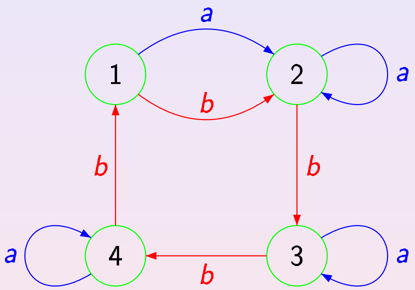

Here one sees 4 **states** called 1,2,3,4, an action called *a* and another action called *b*.

6. Definitions and Terminology

We consider complete deterministic finite automata:

$$\mathscr{A} = \langle Q, \Sigma, \delta \rangle.$$

Here

- Q is the state set;
- Σ is the input alphabet;
- $\delta : Q \times \Sigma \to Q$ is the transition function.

We need neither initial nor final states.

Σ^* stands for the set of all words over Σ including the empty word.

The function δ uniquely extends to a function $Q \times \Sigma^* \to Q$ still denoted by δ.

To simplify notation we often write $q \,.\, w$ for $\delta(q, w)$ and $P \,.\, w$ for $\{\delta(q, w) \mid q \in P\}$.

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7. Definitions and terminology

An automaton $\mathscr{A} = \langle Q, \Sigma, \delta \rangle$ is called synchronizing if there exists a word $w \in \Sigma^*$ whose action resets $\mathscr{A}$, that is, leaves the automaton in one particular state no matter which state in Q it started at: $\delta(q, w) = \delta(q', w)$ for all $q, q' \in Q$.

We can also write this as $|Q\,.\,w| = 1$.

Any word w with this property is a reset word for $\mathscr{A}$.

Other names:

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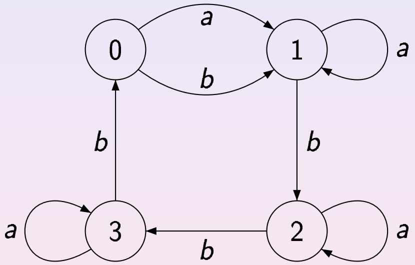

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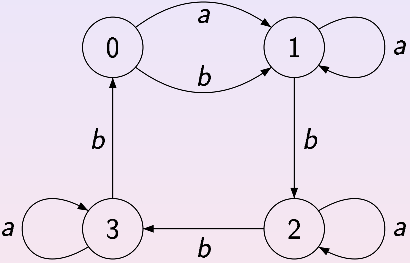

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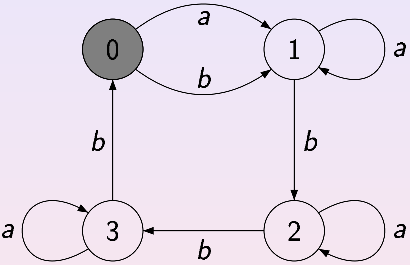

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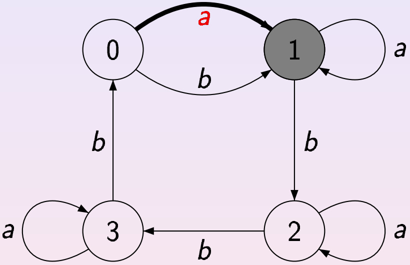

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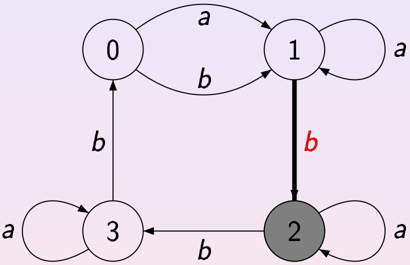

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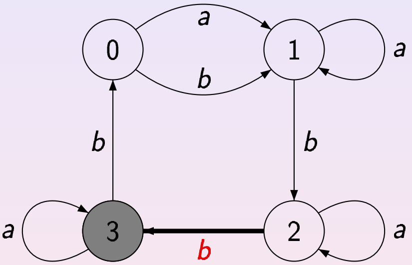

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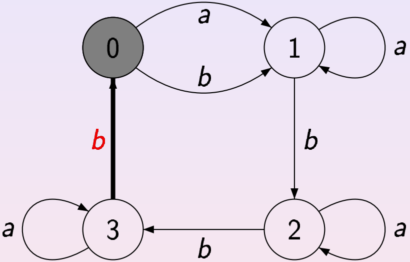

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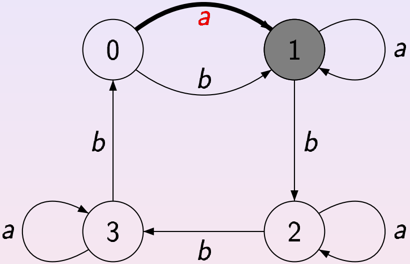

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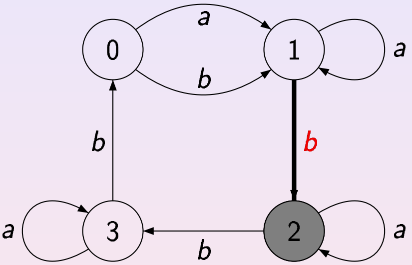

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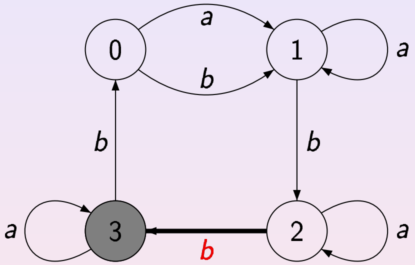

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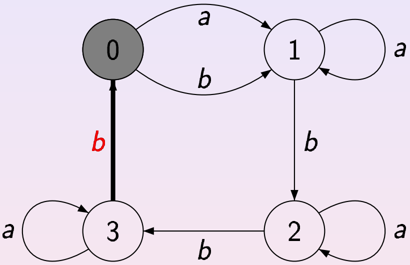

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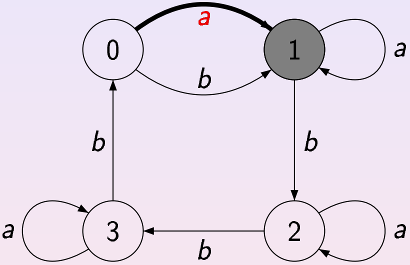

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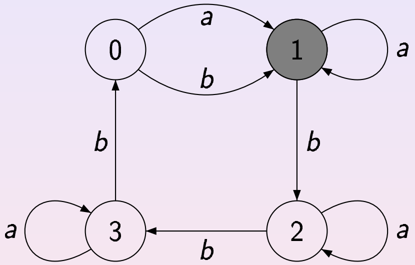

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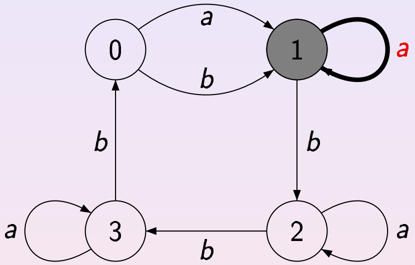

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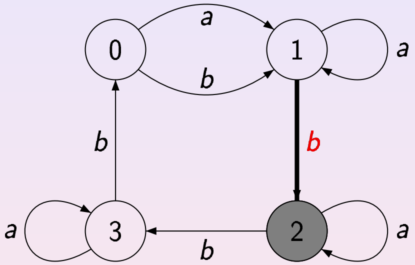

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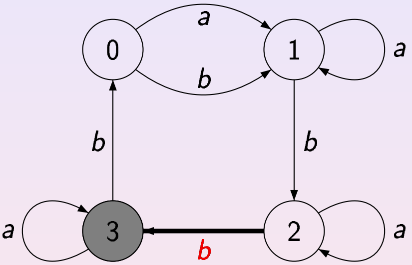

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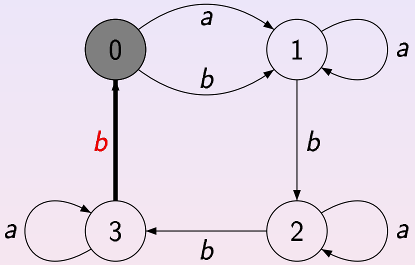

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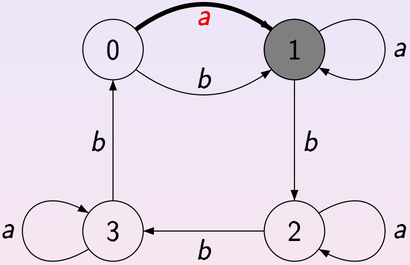

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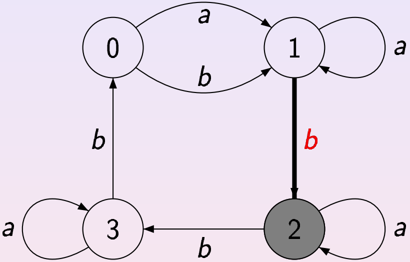

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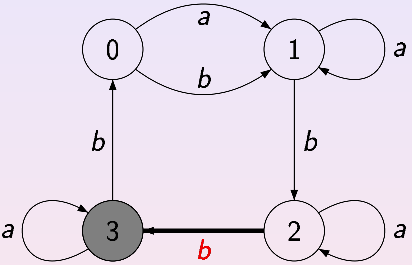

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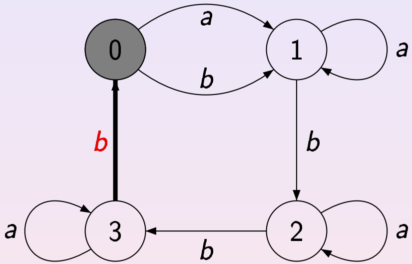

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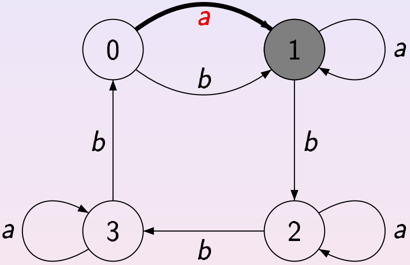

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The idea of synchronization is pretty natural and of obvious importance: we aim to restore control over a device whose current state is not known.

Think of a satellite which loops around the Moon and cannot be controlled from the Earth while "behind" the Moon (Černý's original motivation).

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10. Other Sources

It is not surprising that synchronizing automata were re-invented a number of times:

- The notion was very natural by itself and fitted fairly well in what was considered as the mainstream of automata theory in the 1960s.
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11. Crash Course in Coding Theory

A prefix code over a finite alphabet Σ is a set X of words in Σ^* such that no word of X is a prefix of another word of X. A prefix code is maximal if it is not contained in another prefix code over the same alphabet. A maximal prefix code X over Σ is synchronized if there is a word $x \in X^*$ such that for any word $w \in \Sigma^*$, one has $wx \in X^*$. Such a word x is called a synchronizing word for X.

The advantage of synchronized codes is that they are able to recover after a loss of synchronization between the decoder and the coder caused by channel errors.

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12. Synchronized Codes

$\Sigma = \{0, 1\}$, $X = \{000, 0010, 0011, 010, 0110, 0111, 10, 110, 111\}$.
Then X is a maximal prefix code and one can easily check that each of the words 010, 011110, 011111110, . . . is a synchronizing word for X.

The vertical lines show the partition of each stream into code words and the boldfaced code words indicate the position at which the decoder resynchronizes.

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Sent	0 0 0 | 0 0 1 0 | 0 1 1 1 | . . .
Received	1 0 0 0 **0 1 0** 0 1 1 1 . . .
Decoded	1 0 | 0 0 0 | 1 0

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If X is a finite maximal prefix code, then its decoding can be implemented by a DFA.

Synchronized codes precisely correspond to synchronizing automata!

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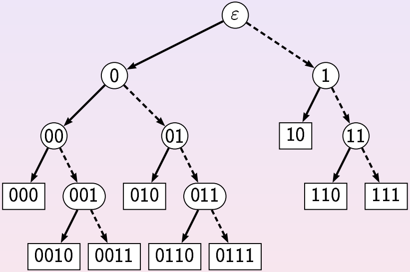

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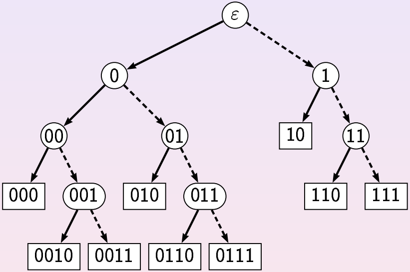

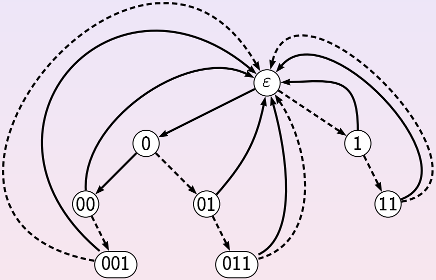

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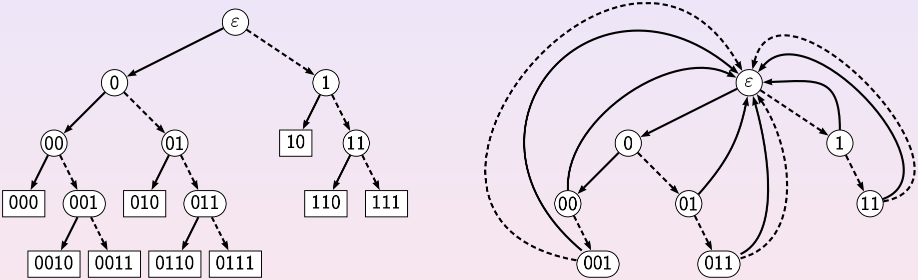

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14. Re-inventing by Engineers

Since the 60s synchronizing automata have been considered as a useful tool for testing of reactive systems (first circuits, later protocols) and have been also applied in coding theory.

In the 80s, the notion was reinvented by engineers working in a branch of robotics which deals with part handling problems in industrial automation.

Suppose that one of the parts of a certain device has the following shape:

Such parts arrive at manufacturing sites in boxes and they need to be sorted and oriented before assembly.

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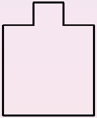

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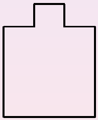

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Assume that only four initial orientations of the part shown above are possible, namely, the following ones:

Suppose that prior the assembly the part should take the 'bump-left' orientation (the second one in the picture). Thus, one has to construct an orienter which action will put the part in the prescribed position independently of its initial orientation.

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Suppose that prior the assembly the part should take the 'bump-left' orientation (the second one in the picture). Thus, one has to construct an orienter which action will put the part in the prescribed position independently of its initial orientation.

We put parts to be oriented on a conveyer belt which takes them to the assembly point and let the stream of the parts encounter a series of passive obstacles of two types (*high* and *low*) placed along the belt.

A high obstacle is high enough so that any part on the belt encounters this obstacle by its rightmost low angle.

Being curried by the belt, the part then is forced to turn 90° clockwise.

16. Re-inventing by Engineers

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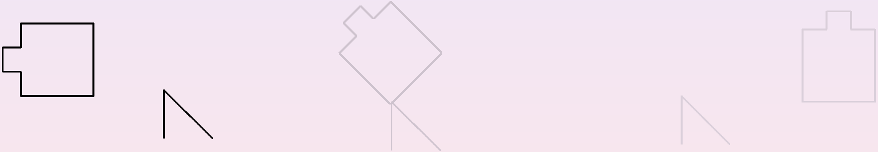

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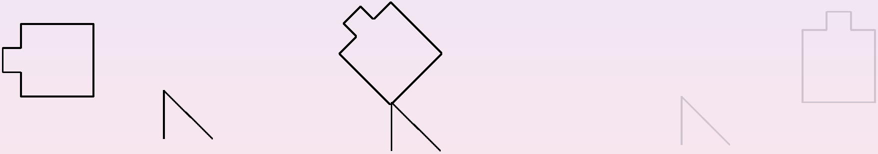

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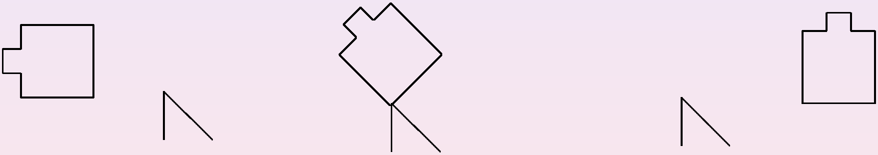

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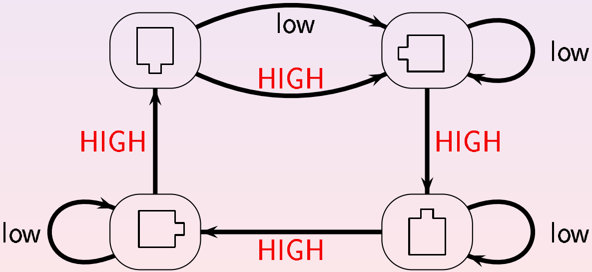
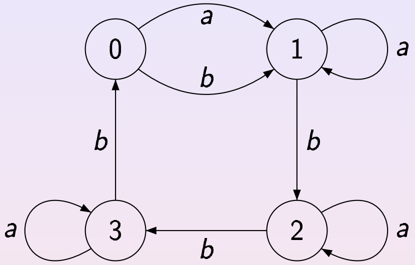
17. Re-inventing by Engineers

A low obstacle has the same effect whenever the part is in the "bump-down" orientation; otherwise it does not touch the part which therefore passes by without changing the orientation.

The following schema summarizes how the obstacles effect the orientation of the part in question:



We met this picture a few slides ago:



– this was our example of a synchronizing automaton, and we saw that *abbbabbba* is a reset sequence of actions. Hence the series of obstacles

low-HIGH-HIGH-HIGH-low-HIGH-HIGH-HIGH-low

yields the desired sensorless orienter.

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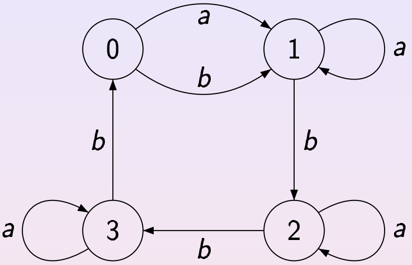

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19. Possible Use in Biocomputing

In DNA-computing, there is a fast progressing work by Ehud Shapiro's group on "*soup of automata*" (Programmable and autonomous computing machine made of biomolecules, Nature 414, no.1 (November 22, 2001) 430–434; DNA molecule provides a computing machine with both data and fuel, Proc. National Acad. Sci. USA 100 (2003) 2191–2196, etc).

They have produced a solution containing 3×10^{12} identical DNA-based automata per μl. These automata can work in parallel on different inputs (DNA strands), thus ending up in different and unpredictable states. One has to feed the automata with an reset sequence (again encoded by a DNA-strand) in order to get them ready for a new use.

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20. Outline of this Course

- From the viewpoint of applications, real or yet imaginary, algorithmic issues are of crucial importance.
- Synchronizing automata constitute an interesting combinatorial object. Their studies from a combinatorial viewpoint are mainly motivated by the Černý Conjecture.
- Interesting connections to symbolic dynamics have led to the Road Coloring Problem.
- We present in detail a recent proof of the Černý Conjecture for the special case of aperiodic automata.
- There are also interesting connections with the Perron–Frobenius theory of non-negative matrices.
- We also formulate several tantalizing open problems.

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